INVESTIGATION OF THE HYDRODYNAMIC BOUNDARY LAYER ON A DISK WASHED BY A TRANSVERSE STREAM

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The results are given of an experimental investigation and of theoretical calculation of the hydrodynamic boundary layer on flat disks washed by transverse flow of an air stream.

The literature contains few investigations devoted to the problem under consideration. There are solutions [1, 2] dealing with analysis of perpendicular flow of an ideal fluid over a disk. However, in view of the appreciable simplifications in the original model of the flow, the results of the solution in [1] turn out to be quite unsuitable, while the results of the solution in [2] are unsuitable in the peripheral region of the disk (see Fig. 3). There has been no success in obtaining a more accurate solution, for which reason a purely theoretical investigation of the given problem is not possible, and it was necessary to continue on the basis of an experimental relationship, which has been obtained also in the present work.

As will be seen below the experimental relationship has been obtained in the form of an odd power series of degree nine. The high degree of the series prevented the author from computing the boundary layer by the exact methods of Blasius-Howarth [3] or Görtler-Witting [14]. In the circumstances he had to use one of the approximate methods of calculation based on integral relations. This was justified by the fact that, as was pointed out in [3, 4], for convergent flow the approximate methods give satisfactory results almost coinciding with the exact solution.

Apropos of the use in calculation of an approximate relation and of a number of the other approximations there is interest in a final experimental verification of results of calculation by comparison of the calculated and experimental velocity profiles in the boundary layer, this being also an objective in the present paper.

Experimental Equipment, Test Conditions, and Method of Measurement. The experimental equipment (Fig. 1) consisted of a low-turbulence wind tunnel, the test models, instruments, and devices for defining the coordinates and measuring the magnitude and direction of the local flow velocity.

The straight-through wind-tunnel was designed and built in accordance with the basic recommendations of experimental aerodynamics, thus achieving a good quality of flow and reliable measurement of its parameters. Special attention was given to means of reducing turbulence and other flow fluctuations, which dictated a requirement firstly to express the influence of the Reynolds number in "intrinsic" form or to approximate to this, and secondly, to increase the accuracy of measurement.

Nozzle 1, of square section and giving a compression of 6.6, had a Vitoshinskii profile and followed the ratios of dimensions indicated in [5]. Two anti-turbulence grids 21, with characteristics recommended in [6], were mounted in series in the nozzle entrance section.

The working section 2 was of closed type and square section, measuring 350 × 350 mm, and was equipped with longitudinal (18) and transverse (19) reference scales and reading glasses.

The turning section 5 was equipped with a vane system 6. The first (7) and second (10) diffusers had an opening angle of 6 degrees. A honeycomb 13 was located ahead of the fan 9. The great elasticity and sponginess of the soft mounting 8, in addition to isolating the working section and the instruments from fan vibration, permitted realization of the recommendations of [7] regarding reduction of aerodynamic and acoustic fluctuations. The fan was centrifugal and had vibration-isolation mounting. Control of the flow velocity in the tunnel was effected by varying the revolution rate of the fan. The distributor (damper) 11 was of special construction to achieve uniform flow distribution and was made of wire mesh in the form of a cylindrical chamber with three mesh partitions. In accordance with the recommendations of [8], the wind tunnel was located in a room with large volume (300 m^3), and to secure greater damping of the air returning to the nozzle, the room was partitioned off by the gauze screen 12.

Determination of the flow velocity in the tunnel was made from the static pressure drop between two sections of the nozzle. The take-off of static pressures was accomplished by means of four nipples located around the perimeter of each section joined via the common ring collector 20. The pressure drop was measured with a compensated micromanometer 19. Taking into account the comments in [9] concerning the factors which decrease the accuracy of readings of the compensated micromanometer, it was specially adjusted, and thus an accuracy of ±0.2 N/m^2 was achieved. Because of its relatively large inertia, the compensated micromanometer was insensitive to velocity fluctuations in the wind tunnel that are observed on an inclined manometer, and indicated stable values of mean velocity. A preliminary calibration of the tunnel was done observing the recommendations of [6]. The maximum flow velocity in the empty tunnel was 14 m/sec. The non-uniformity of velocity field in the central core of the working section measured by the technique described in [10] was 0.985. The level of turbulence of the flow in the working section as measured by means of a thermal anemometer with an amplifier did not exceed 0.2%.

The experimental models used were flat polished cylindrical disks with a sharp rectangular edge. The

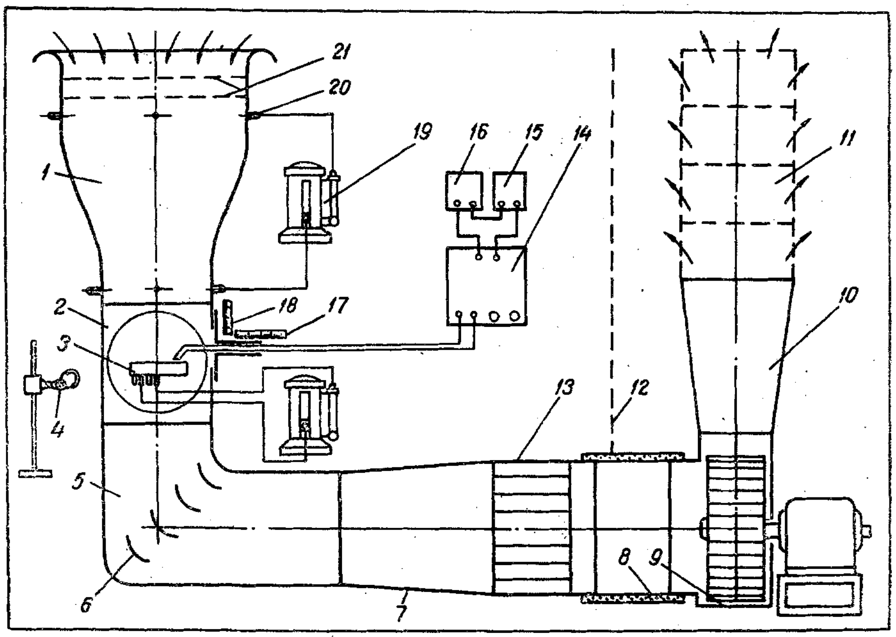

Fig. 1. Schematic of the experimental equipment.

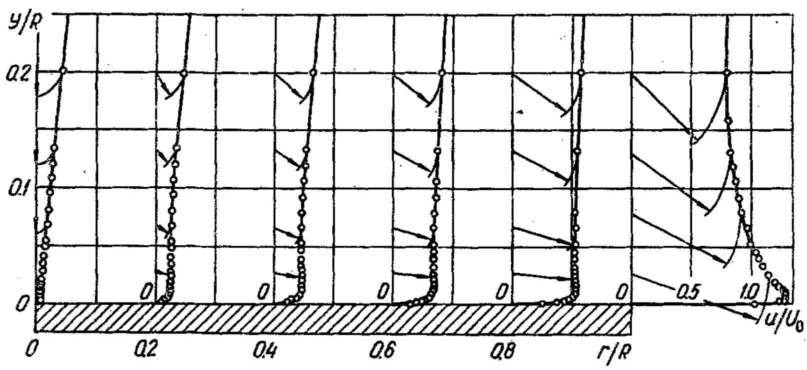

Fig. 2. Dimensionless velocity field above a disk washed by a perpendicular stream; R = 0.075 m, U_∞ = 13.65 m/ /sec. The graphs show the variation of the modulus of the dimensionless velocity.

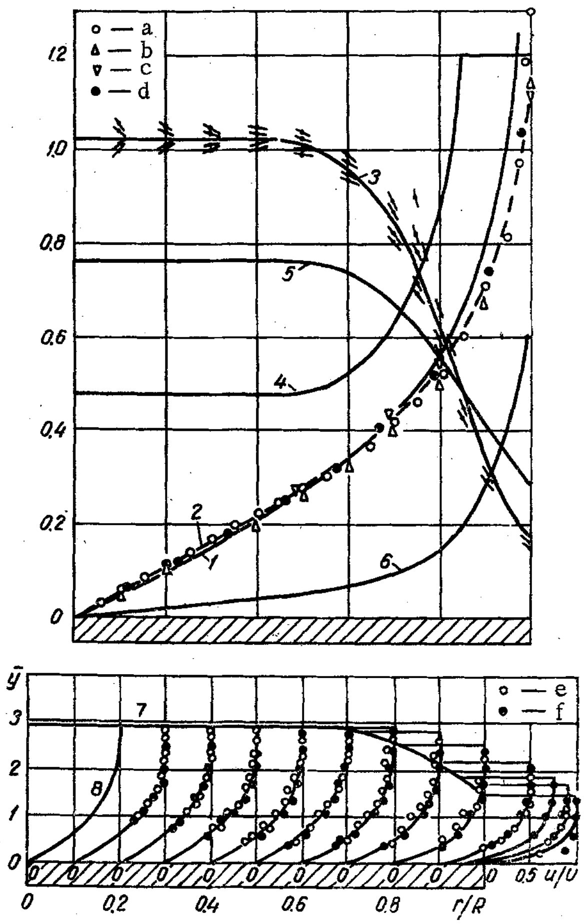

Fig. 3. Results of the experimental investigation and of the theoretical calculation of the hydrodynamic boundary layer on a flat disk with perpendicular flow: the points show the experimental data, the dotted lines are approximations to the experimental data, and the full lines are theory; 1) distribution of $\bar{U}$ at the outer edge of the boundary layer according to [2]; 2) the same, approximations according to formula (1), for the test data; a) the same, measurements with the thermoanemometer for R = 0.075 m, U_∞ = = 13.65 m/sec; b) the same for R = 0.050 m, U_∞ = 13.4 m/sec; c) the same for R = 0.025 m, U_∞ = 12 m/sec; d) the same relevant to pressure distribution for R = 0.0445 m, U_∞ = 12 m/sec; 3) to 10) $\bar{Z}$, computed from equation (2) by means of isoclines; 4) 0.1 λ; 5) δ*; 6) 0.1 c_f; 7) $\bar{\delta}$; 8) distribution of velocities in the boundary layer, according to theory; e) the same according to experiment for Re = $4 \cdot 10^4$; f) the same according to experiment for Re = $7 \cdot 10^4$.

disks were 30 mm thick and of diameter 30, 40, 50, 89, 100, and 150 mm. In order to evaluate the effect of disk thickness on the velocity field, one of the disks of 100-mm diameter was made with a sharp conical edge and mounted with its larger end pointing upstream. The flow over this type of body was equivalent to flow over a thin disk. The surface of disk 3 (of 89-mm diameter) was drilled with holes of diameter 0.5 mm, positioned along a radius on a pitch of 5 mm.

Bearing in mind that it was required to make measurements in a boundary layer of thickness ranging from one to several tenths of a millimeter, and this in conditions where the direction of the velocity vector underwent marked change, a thermoanemometric method was chosen for measurement of velocity distribution in the boundary layer. The choice was dictated by the small dimensions of the working element of the thermoanemometer 14 (in the form of a wire of 20μ-diameter and 3-mm length), and by the fact that in the axisymmetrical conditions of the flow one may keep the element perpendicular to the local velocity direction and measure the velocity amplitude. The measurements were conducted in the constant temperature mode. For measurements above the central part of the disk surface, in conditions of substantial velocities of the oncoming stream, it proved to be impossible to use in the thermoanemometer circuit the usual low-inertia galvanometer 16, owing to the strong non-uniform fluctuations in the local velocity. In these circumstances the galvanometer 15 was used for which the inertia was increased by filling its cavity with transformer oil. There was no noticeable change in its sensitivity in doing this. Near the surface the thermoanemometer readings were distorted. The distortions were taken into account to some degree by means of corrections according to the method described in [11]. There was no change in the radiant component in going from calibration to the tests, nor during the tests since the wire temperature is kept constant in this mode of operation of the thermoanamometer, and the degree of emissivity was practically unchanged because of the very small surface area of the working element.

The direction of the velocity vector was determined visually from the position of a single-strand silk thread of 1-mm length and diameter about 0.01 mm, attached so as to be free to pivot on the wire of the thermoanemometer head. The angle of slope of the thread was measured with the binocular loop 4 of type BL-2 provided for this purpose. The measurements were made by bringing a reference line at the focus of the ocular into coincidence with the thread in the anemometer head. The direction was read from an angle scale with 1° divisions.

In the tests the range of velocity was 2 to 14 m/sec and of disk radius 0.015 to 0.075 m, corresponding to a Reynolds number range of $2 \cdot 10^3$ to $7 \cdot 10^4$.

Results of Measurements. The test results were used to construct graphs of dimensionless velocity amplitude (Fig. 2), from which the velocity distribution at the edge of the boundary layer (a, b, and c on Fig. 3) was determined, as well as velocity profiles of its internal flow (e and f on Fig. 3). It is clear from the graphs of dimensionless velocity that there is a region above the boundary layer in each section of which the vector velocity remains constant in absolute value. These values were assumed also to be those for potential flow at the edge of the boundary layer. The correctness of this procedure was verified by a check measurement of pressure distribution above the surface of the drilled disk (d on Fig. 3). A more accurate measurement of the angle of slope of the velocity vector proved to be difficult because of the low inherent accuracy of the method of measurement. It was established, however, that near the boundary layer edge the slope angles of the velocity vector relative to the surface do not exceed a few degrees and we therefore take the absolute value of velocity to be the parallel component of the stream velocity at the edge of the boundary layer to an accuracy sufficient for the present purpose.

The measurements of velocity field above the disk of 100-mm diameter with a sharp conical edge showed no noticeable differences in comparison with the velocity field above the cylindrical disk. This shows that the effect of disk thickness, if present, is inappreciable.

During the measurements an effect of disk diameter on the dimensionless velocity field was observed, connected with the finiteness of the stream in the wind tunnel. It was worthy of note that this effect was in evidence only in the peripheral part of the disks, disappearing asymptotically as their diameter decreased. This factor was taken into account in correlating the results in that experimental points noticeably affected by this factor were not taken into consideration.

Making a comparison of the experimentally obtained dimensionless velocity fields among themselves and with the theoretical solution over disks of arbitrary diameter, it may be considered that the dimensionless velocity distribution at the outer edge of the boundary layer on a disk washed perpendicularly is universal, depending only on the dimensionless radius $\bar{r}$. This distribution is approximated satisfactorily by the expression

$$\bar{U} = 0.56\bar{r} + 0.56\bar{r}^9, \qquad (1)$$

represented by Curve 2 on Fig. 3.

Theoretical Calculation. Calculation of the boundary layer on the basis of relation (1) was accomplished by the Karman-Polhausen approximate method in the form in which it has been developed by Holstein and Bohlen [3]. It was convenient to use the relations in universal form, so that the dimensionless quantities $\bar{r}$, $\bar{U}$ and $\bar{U}'$ can be used directly in the calculation, and the computational characteristics of the boundary layer can be obtained in dimensionless form. Under these requirements the momentum equation for the axisymmetric boundary layer on the flat disk may be written as

$$\frac{d\bar{Z}}{d\bar{r}} = \frac{1}{\bar{U}}\left[F(\varkappa) - \frac{2\varkappa\bar{U}}{\bar{r}\bar{U}'}\right],$$
$$\bar{Z} = \bar{\vartheta}^2, \ \varkappa = \bar{Z}\bar{U}'. \qquad (2)$$

At the stagnation point $\overline{Z}_0 = \varkappa_0/\overline{U}'_0$, $\varkappa_0 = 0.05708$.

It is customary to integrate Eq. (2) by approximate methods: graphico-analytically—by isoclines, or analytically—by reduction to a quadrature by approximating the function $F(\varkappa)$ by a linear relation. Concerning integration in a region of strong pressure reduction, the author of [3] recommends use of the second method. However, reduction to a quadrature requires prior knowledge of the final values of the function $F(\varkappa)$ or of its argument in the range of which it is subject to linearization. In the case under examination only the value of the shape factor $\varkappa_0$ at the stagnation point is known. It is therefore improper to use the analytical method. As regards the isocline method, it is noted in [3] that it is unsuitable in a region of strong pressure fall, after the form factors reach value of $\varkappa = 0.0948$ ($\lambda = 12$) since the function $F(\varkappa)$ becomes double-valued for one thing. In these circumstances the following procedure was used. It is pointed out in [3] that in an accelerated establishment of flow, the assumed one-parameter profiles in the Karman-Polhausen method, in the form of a polynomial of fourth degree, may experience deformation, according to the monotonic requirement, only to a definite state characterized by shape factors $\lambda = 12$ and $\varkappa = 0.0948$ since the profiles corresponding to $\lambda > 12$ are improbable. On this basis the isocline field was constructed according to Eq. (2) in the range only up to $\lambda = 12$, ($\varkappa = 0.0948$, i.e., isoclines corresponding to $\lambda > 12$ were not taken into account.

As a result of the analysis of the isocline field, constructed within the assumed limits of variation of the shape factor, it turned out, of course, that the relation $\varkappa = \overline{Z}\overline{U}'$, corresponding to the behavior of the curve $\varkappa(\lambda)$, from Eq. (2) and taking the form

$$\overline{Z} = 0.0948/\overline{U}', \tag{3}$$

at the maximum value $\varkappa = 0.0948$, is a limit for existence of a solution of the differential Eq. (2). Careful construction of the desired integral curve from the isoclines has shown that, on attaining the value $\varkappa = 0.0948$, it falls smoothly towards the boundary for existence of the isocline field, merges smoothly, and thereafter coincides with it. The latter circumstance means that, after attaining $\varkappa = 0.0948$ (in the case considered this is approximately at $r = 0.85$), the desired solution coincides with relation (3), and leads to constancy of the shape factors at $\varkappa = 0.0948$ ($\lambda = 12$) throughout the entire final section. The physical meaning of this result is that if, during convergent flow, a state is attained by the profile corresponding to $\lambda = 12$, and if no factors arise subsequently that may induce an inverse deformation, the profile attained must be conserved in the later flow.

We note that, as is to be expected, the method of reduction to quadrature recommended in [3] for a region of strongly of strongly falling pressure leads to the same constancy of shape factors, if linearization of the function $F(\varkappa)$ is performed in the range $0.0570 \leq \varkappa \leq 0.0948$ quoted in the previous calculation, the reason being that at $\varkappa = 0.0948$ the approximating function coincides with that being approximated.

The remaining characteristics of the boundary layer (λ, $\overline{\delta}$, $\overline{\delta}^*$, c_f and u/U), as calculated by well-known relations from the graph of $\overline{Z}$ (Fig. 3, Curve 3), are shown on the same figure (Curves 4–8).

Summarizing the results obtained, it may be concluded, that in the range examined, the distribution of all the dimensionless characteristics of the boundary layer on a disk washed perpendicularly are universal depending only on the dimensionless radius. It is evident from Fig. 3 that the peculiarities of the critical point region, the linear growth of velocity at the outer edge of the boundary layer, and of the shear stress at the wall, the constancy of the boundary layer thickness, and the affinity of the velocity profiles of its internal flow are conserved in practically the entire central region of the disk up to $\overline{r} = 0.5$. Later the growth of velocity and of shear stress increases rapidly, the thickness of the boundary layer thickness of the boundary layer diminishes, and the velocity profiles change shape sharply, after which no noticeable change occurs in the whole peripheral region of the disk, up to the edge itself. The integral curve of $\overline{Z}$ (Fig. 3, Curve 3) is approximated satisfactorily by the expression

$$\overline{Z} = 0.1019\,(1 - 8\overline{r}^{8} + 7.166\,\overline{r}^{9}), \tag{4}$$

from which all the characteristics of the boundary layer required in practice may be calculated from well-known relations.

The nature of the profiles is evidence that the structure of the internal flow in the boundary layer was laminar in the whole range of investigation. Judging from the profiles it may be concluded that there is no separation of the laminar boundary layer in conditions where disks are washed by perpendicular flow.

NOTATION

R is the disk radius; r is the current radius; $\overline{r} = r/R$ is the same in dimensionless form; U_∞ is the velocity of the oncoming stream; u is the local velocity; U is the velocity at the outer edge of the boundary layer; $\overline{U} = U/U_\infty$ is the same in dimensionless form; U' is the first derivative of U with respect to r; $\overline{U}'$ is the first derivative of $\overline{U}$ with respect to $\overline{r}$; ν is the kinematic viscosity; y is the coordinate along the normal to the disk; $\overline{y} = (y/R)(U_\infty R/\nu)^{1/2}$ is the same in dimensionless form; δ is the thickness of the boundary layer; $\overline{\delta} = (\delta/R)(U_\infty R/\nu)^{1/2}$ is the same in dimensionless form; δ^* is the displacement thickness; $\overline{\delta}^* = (\delta^*/R)(U_\infty R/\nu)^{1/2}$ is the same in dimensionless form; ϑ is the momentum loss thickness; $\overline{\vartheta} = \vartheta/R\,(U_\infty R/\nu)^{1/2}$ is the same in dimensionless form; ρ is the density; τ_0 is the shear stress at the wall; $c_f = (2\tau_0/\rho U_\infty^2)\cdot(U_\infty R/\nu)^{1/2}$ is the same in dimensionless form; $\lambda = U'\delta^2/\nu$ is the first shape factor; $\varkappa = U'\vartheta^2/\nu$ is the second shape factor; $F(\varkappa)$ is the auxiliary function whose values are given in Table 14 and in Fig. 106 in [3]; $Re = U_\infty R/\nu$.

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